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ABSTRACT

A Project Solo module on communication matrices is presented which uses a fictitious airline to demonstrate the principles of communication patterns. It is noted that the Project Solo curriculum is designed to allow for several points of entry and a multiplicity of paths through the modules. (JY)

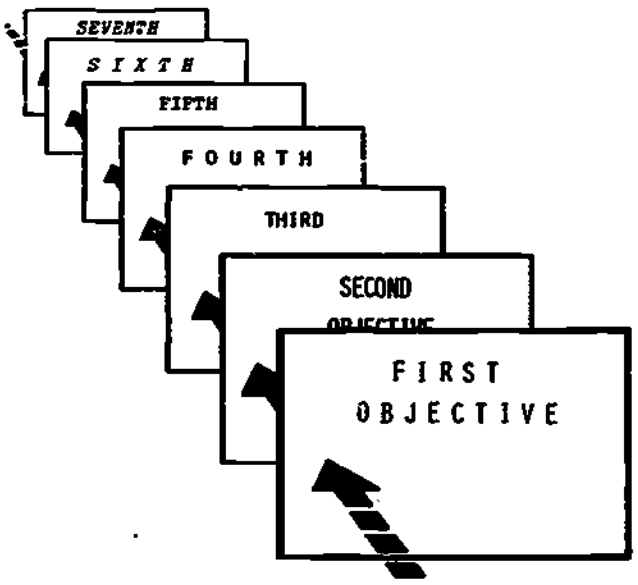
PROJECT SOLO

AN EXPERIMENT IN REGIONAL COMPUTING FOR SECONDARY SCHOOL SYSTEMS

University of Pittsburgh • Department of Computer Science • Pittsburgh, Pennsylvania 15213

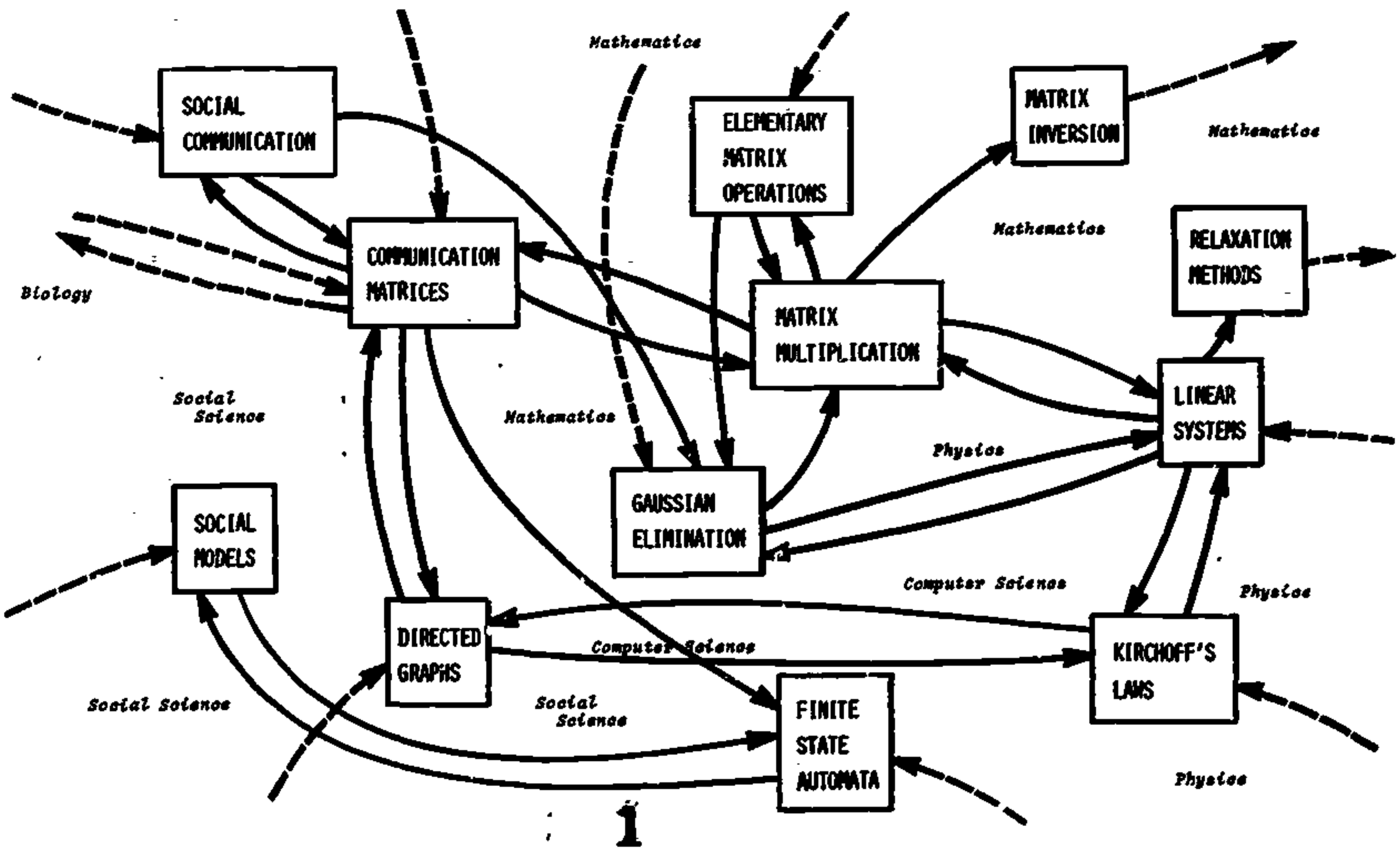
Newsletter No. 17 June 14, 1971

A number of "individualized learning" schemes are built around the premise that a given body of knowledge is best organized as a sequence of precise learning objectives. The student does not move to a given objective in this sequence until he has mastered all previous objectives.



Silberman[1] has pointed out one objection to such a scheme in writing "it does not permit any position between the two extremes [total prescription of goals by others, total freedom for the student to pursue his own goals].... The instructional technology can work only if those who write the programs specify every goal in the most precise and detailed terms.... The system cannot accomodate a student who wants to strike out on his own."

When one considers the variation in personalities and learning styles of individual learners, it makes more sense to organize the elements of a curriculum in a scheme that allows for several entry points, and a multiplicity of paths. Consider the example:



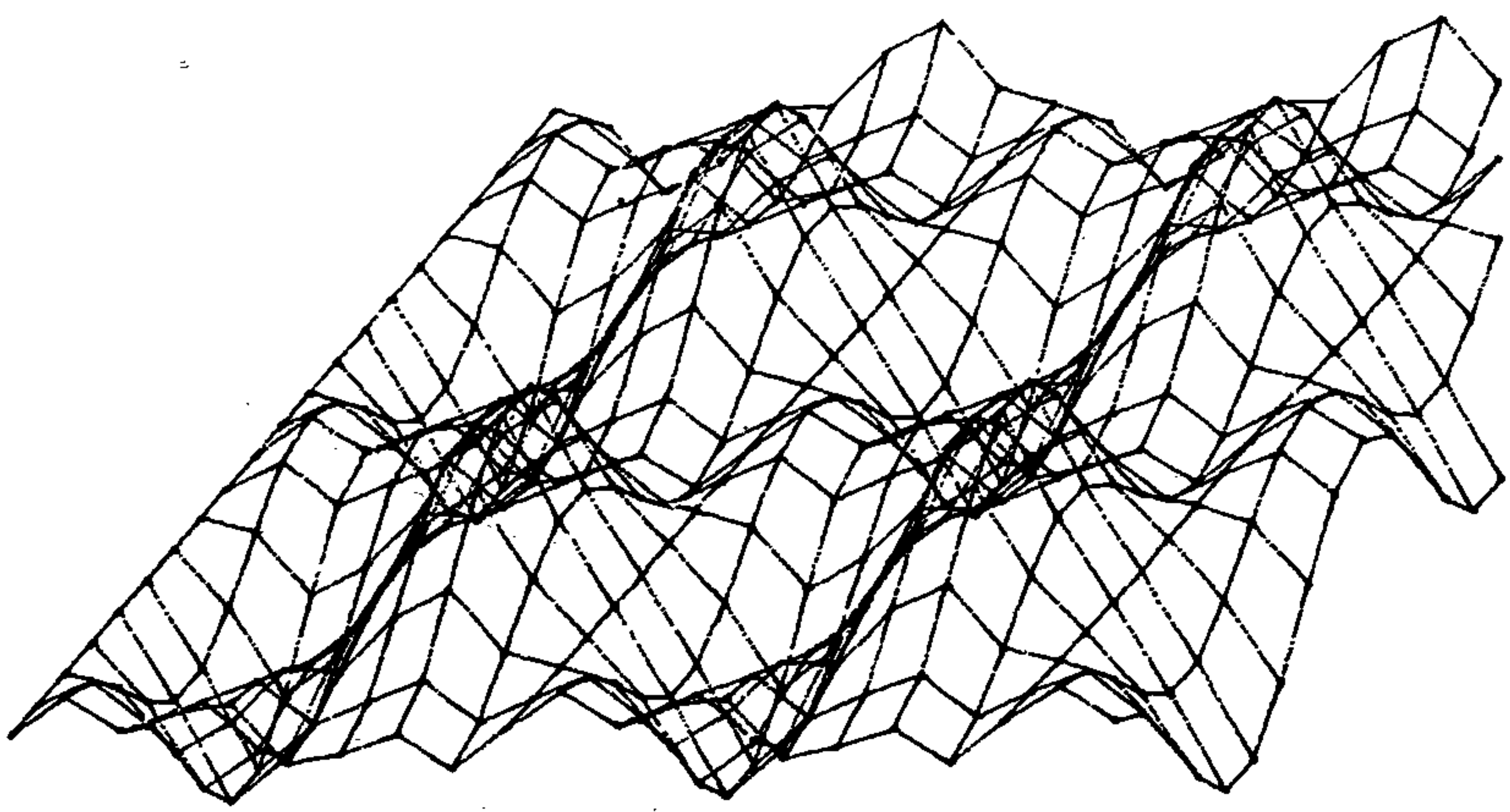
Most of the modules of Project Solo have been written with such a scheme in mind. As can be seen in the example diagram, several disciplines "feed" each other in various ways, illustrating the fact that a relatively small number of process skills can serve a student well, especially if he is aware of these relations. Equipping learners with a global outlook of this kind is certainly one of the most challenging goals of education.

Communication Matrices

A copy of the module "Communication Matrices" is enclosed. Comments, corrections, suggestions, etc. should be sent to Emilie Zielinski.

From the Plotter Waste Basket

The picture below is offered without explanation. However you might be interested in knowing that Frank Wimberly is working on some modules that explore the subject of "linear perspective". These can serve geometry, art, or any science that is interested in 3-D graphs. Computer generated movies are another application of such techniques.



[1]Silberman, Charles E. Crisis in the Classroom. Random House. (1970).

*Supported in part by NSF Grant GJ-1077

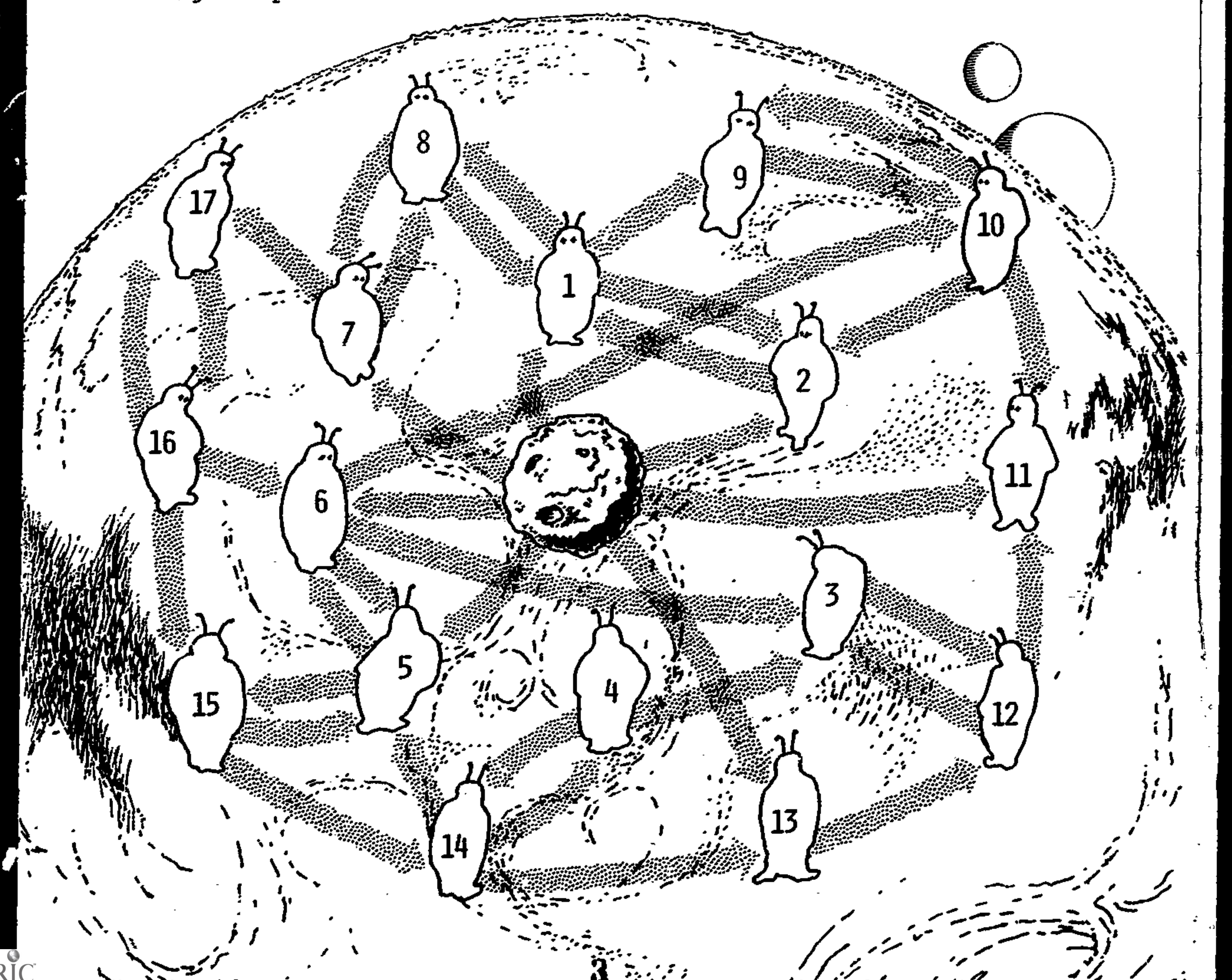
Communication Matrices

PROJECT SOLO
Department of Computer Science, University of Pittsburgh
Pittsburgh, Pennsylvania (15213) Module 0045

(FEATURING THE CASE OF THE MUTATED VIRUS ON PLANET X!)

● This module shows the application of matrices in studying communication patterns. Problems related to airline ticketing, message delivery, and communicable diseases will be presented.

● The module "Matrix Multiplication" is a prerequisite for doing the problems in this module.



The Airline Ticketing Problem:

EAGLE Airlines, a little known but ambitious "scheduled" commuter service, lists the following direct flights:

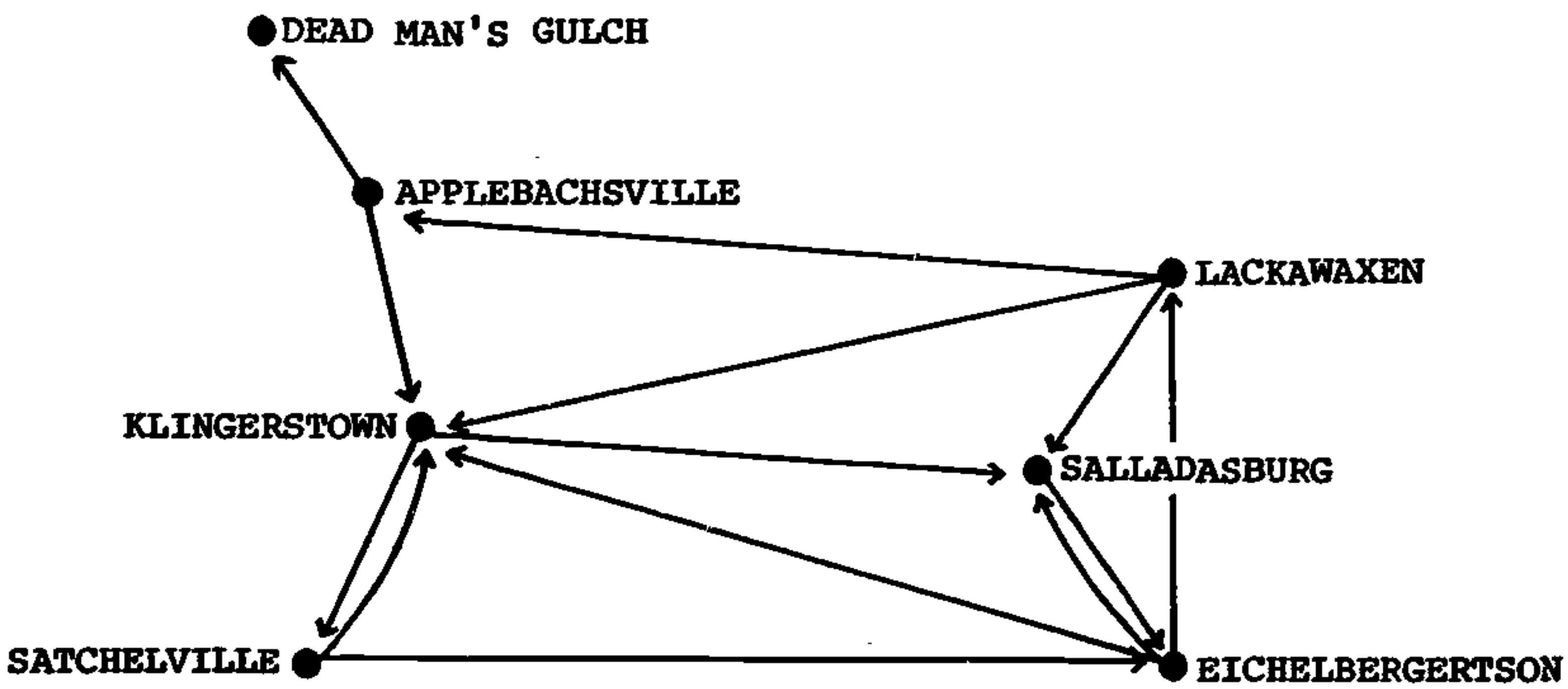
Eichelbergertown	to	Salladasburg
Eichelbergertown	to	Klingerstown
Eichelbergertown	to	Lackawaxen
Applebachsville	to	Dead Man's Gulch
Applebachsville	to	Klingerstown
Lackawaxen	to	Applebachsville
Lackawaxen	to	Klingerstown
Lackawaxen	to	Salladasburg
Satchelville	to	Klingerstown
Satchelville	to	Eichelbergertown
Salladasburg	to	Eichelbergertown
Klingerstown	to	Satchelville
Klingerstown	to	Salladasburg

An interesting question we can ask is the following:

> For any given starting city, how many different ways can you fly to each of the cities (including the starting city) via EAGLE Airlines?

(We are not concerned with how long a passenger may have to wait for a connection in this problem).

Diagrammatically we can represent EAGLE's flight schedule as shown below:



A diagram such as the above is called a directed graph. The direction of the arrows represents a flight from one town to another.

(NOTE: We're assuming that there are no flights scheduled that take off, circle overhead, then land in the very same town! Such a connection would be represented as:

Such connections are useful in some applications. After finishing this module, see if you can think of any such applications. Also check the modules on "Finite State Automata.")

Another way to represent the EAGLE flight routes is with a rectangular array:

TO: / FROM	EICHELBERGERTOWN	APPLEBACHSVILLE	LACKAWAXEN	SATCHELVILLE	SALLADASBURG	KLINGERSTOWN	DEAD MAN'S GULCH
EICHELBERGERTOWN	NO	NO	YES	NO	YES	YES	NO
APPLEBACHSVILLE	NO	NO	NO	NO	NO	YES	YES
LACKAWAXEN			NO				
SATCHELVILLE				NO			
SALLADASBURG					NO		
KLINGERSTOWN	NO	NO	NO	YES	YES	NO	
DEAD MAN'S GULCH							

Either "YES" or "NO" is put in each box in answer to the question "Is there a direct flight from (row city) to (column city)?"

A part of the table has been filled in. Complete the table by looking at EAGLES's schedule on page 2, placing a YES or NO in each space.

When using the computer to help solve Problem 1, the table above is represented as a matrix. The rows and columns of the matrix correspond to the rows and columns of the table. The elements of the matrix are 0 to represent NO and 1 to represent YES.

The matrix representation would be:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By representing the flight schedule as this binary matrix we can extract further information about the schedule. The matrix operations of addition and multiplication are used in the investigation.

In the binary matrix the non-zero entries represent a <u>direct</u> connection from one town to another. This direct connection will be called a <u>1-path</u>, or a path of <u>length-1</u>. By squaring this matrix (raising it to the 2nd power i.e. calculating A*A) we produce a matrix whose entries are the number of <u>2-paths</u>, or <u>paths of length 2</u>, from one town to another. Such a path of <u>length 2</u> requires 2 direct connection flights to get to the destination.

Squaring the matrix A gives:

$$A^2 = A*A = \begin{array}{c} \begin{array}{ccccccc} a & b & f & c & e & d & g \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(NOTE: A*A was calculated using the method explained in the module "MATRIX MULTIPLICATION".)

First row of A^2

WHAT DOES THIS NEW MATRIX TELL US?

Look at the first row. The numbers there represent the flight originating from Eichelbergertown. There is one 2-path to each of the towns: Eichelbergertown, Applebachsville, Satchelville and Klingerstown, as follows:

a. Eichelbergertown to Salladsburg to Eichelbergertown

b. Eichelbergertown to Lackawaxen to Applebachsville

c. Eichelbergertown to Klingerstown to Satchelville

d. Eichelbergertown to Lackawaxen to Klingerstown

There are two 2-paths to Salladsburg:

e. Eichelbergertown to Lackawaxen to Salladsburg
Eichelbergertown to Klingerstown to Salladsburg

There are no 2-paths to Lackawaxen (f) or to Dead Man's Gulch (g).

What do you think the entries of the matrix which results from cubing the original matrix represent?

We will use the mathematical notation A^2 to mean A*A, and A^3 to mean A*A*A. Note that A^3 also = A^2*A.

In our example with seven cities (in general n cities) it should be "obvious" that a passenger can get from one city to another city with six (in general n-1) or less direct flights.*
If we also include round trips (that is, trips which return the passenger to his starting city), we should count the paths of length n or less. Thus in Problem 1 we're interested in counting all the paths of length n or less that connect any given starting city with any other city.

* Assuming he can get there at all.

One method of obtaining this information is to look at the diagram on page 3 and count all of the different ways. However, it is very easy to miss some paths. A more sophisticated method is to sum all of the power matrices (1st power, 2nd power, etc.) from the 1st power to the nth power, where n is the number of cities (and therefore the number of rows (or columns) in the binary matrix A). [If you don't remember how to add matrices, work through the module "ELEMENTARY MATRIX OPERATIONS".]

In the mathematical notation this is

$$S = A^1 + A^2 + A^3 + \dots + A^{n-1} + A^n$$

where S is obtained by adding the matrices on the right. The entries in the summation matrix are the <u>total</u> number of paths of any length (1 to n) connecting any two towns.

PROBLEMS FOR COMPUTER SOLUTION

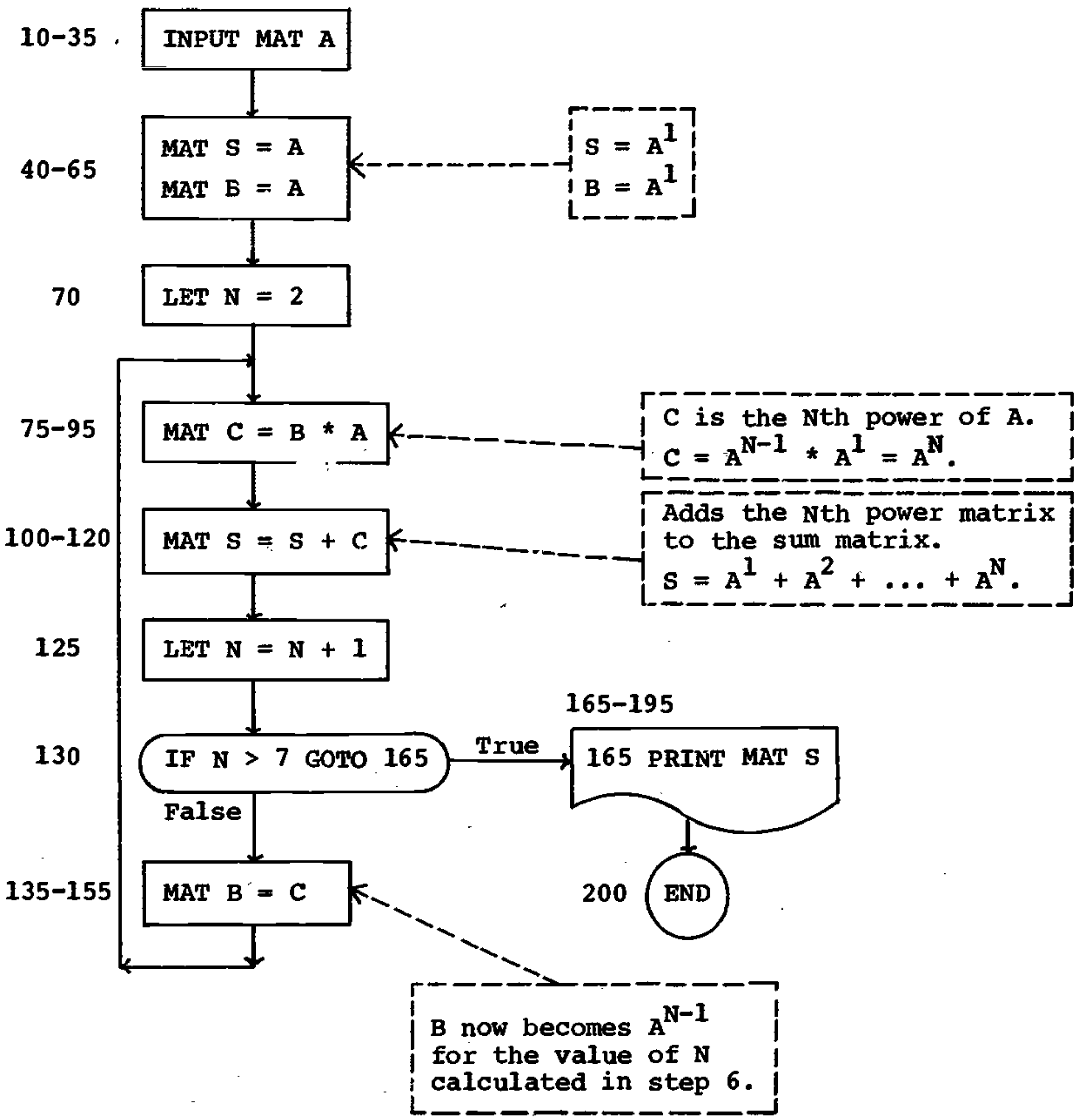
Problem 1

Write a computer program which finds the number of different ways in which a person can fly from a given starting city to each of the other cities (including the starting city) via EAGLE Airlines.

Sample Solution

On page 8 we will show a flow chart describing one solution of this problem; on page 9 a listing of a BASIC program that implements this flow chart is given. This program does <u>not</u> use the matrix functions available in most versions of BASIC in order to show you how a professional programmer manipulates matrices.

Flow Chart, Problem 1



Sample Program

```
5 DIM A(7,7), B(7,7), C(7,7), S(7,7)
10 PR. "INPUT THE CONNECTION MATRIX ROW BY ROW"
15 FOR I= 1 TO 7
20 FOR J= 1 TO 7
25 INPUT A(I,J)
30 NEXT J
35 NEXT I

40 FOR I= 1 TO 7
45 FOR J= 1 TO 7
50 S(I,J) = A(I,J)
55 B(I,J) = A(I,J)
60 NEXT J
65 NEXT I

70 LET N= 2
75 FOR I= 1 TO 7
80 FOR J= 1 TO 7
*85 C(I,J) = B(I,1)*A(1,J)+B(I,2)*A(2,J)+B(I,3)*A(3,J)+B(I,4)*A(4,J)
          +B(I,5)*A(5,J)+B(I,6)*A(6,J)+B(I,7)*A(7,J)
90 NEXT J
95 NEXT I

100 FOR I= 1 TO 7
105 FOR J= 1 TO 7
110 S(I,J) = S(I,J)+C(I,J)
115 NEXT J
120 NEXT I

125 N=N+1
130 IF N>7 THEN 165

135 FOR I= 1 TO 7
140 FOR J= 1 TO 7
145 B(I,J) = C(I,J)
150 NEXT J
155 NEXT I
160 GOTO 75

165 PR. "THE SUMMATION MATRIX IS:"
170 FOR I= 1 TO 7
175 FOR J= 1 TO 7
180 PRINT S(I,J);
185 NEXT J
190 PRINT
195 NEXT I

200 END
```

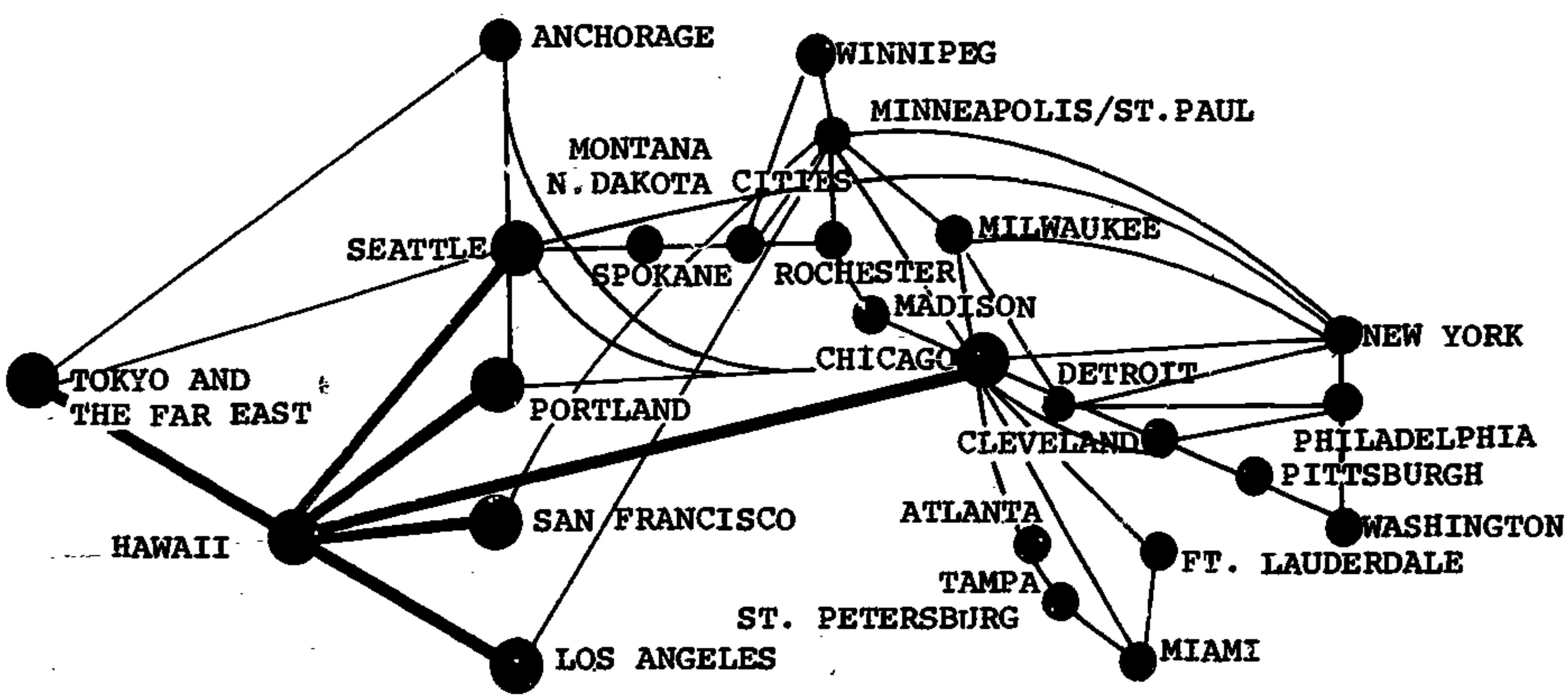
The line numbers used here correspond to those shown on the flow chart of page 8.

*Can you code this step in a more "elegant" manner?

Problem 2

(a) The schedule of a larger airline company is given below. Calculate the number of paths a passenger can choose in travelling from Philadelphia to Hawaii, if he wants to make stops in any three cities before reaching his destination (Hawaii).

(b) On his return, he wishes to go from Hawaii to Washington, again stopping in any three cities before reaching Washington. How many choices does the passenger now have?



Problem 3

Communication matrices can also be used in investigating directed communication of messages.

In the newly formed World Organization Opposed to Polluting the Sea there are eight member countries. In appointing representatives to WOOPS the member countries governing officers overlooked their appointee's language qualifications. Thus each appointee cannot speak to all of the other representatives directly. It is also possible that some representatives are acting under a "closed-mouth" policy. This policy allows a representative to listen to one speaking to him but he, in turn, will not speak or respond to this particular representative.

EXAMPLE:

The Spanish representative speaks to the German representative.

The German representative speaks to the Spanish and the American representatives.

The Italian representative speaks to the American and the French representatives.

The Chinese representative speaks to the Russian representative.

The American representative speaks to the German and the Italian representatives.

The Nigerian representative speaks to the Russian representative.

The Russian representative speaks to the Chinese, the Nigerian, and the French representatives.

The French representative speaks to the Russian representative.

(In this example the French representative is the only one operating under the "closed-mouth" policy.)

Finally, let's assume that a message becomes hopelessly garbled if it is translated more than five times.

Problem:

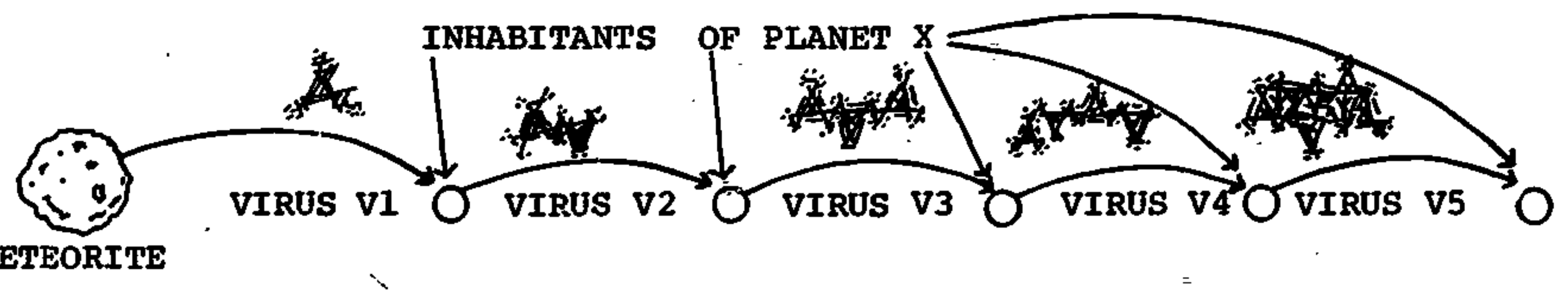
The German representative wishes to argue the validity of throwing empty beer bottles overboard from his country's freighters. Can his statement be routed so that all representatives of WOOPS receive his message? Which countries can and which cannot get messages through to all representatives?

Problem 4

A strange new plague has swept over the countryside of Planet X. It is due to Virus V1, which arrived via a meteorite that has landed from outer space. The inhabitants of Planet X who contacted the meteorite directly contract a disease called D1. To survive they must receive an antidote called A1.

NOW FOR SOME BAD NEWS:

If an inhabitant with disease D1 contacts another inhabitant of X, the virus is transferred in mutated form as V2. The only antidote for V2 is called A2. Similarly, further transfer of the virus produces additional mutations with new antidotes required as follows:



DISEASE	D1	D2	D3	D4	NONE
ANTIDOTE	A1	A2	A3	A4	

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The German representative wishes to argue the validity of throwing empty beer bottles overboard from his country's freighters. Can his statement be routed so that all representatives of WOOPS receive his message? Which countries can and which cannot get messages through to all representatives?

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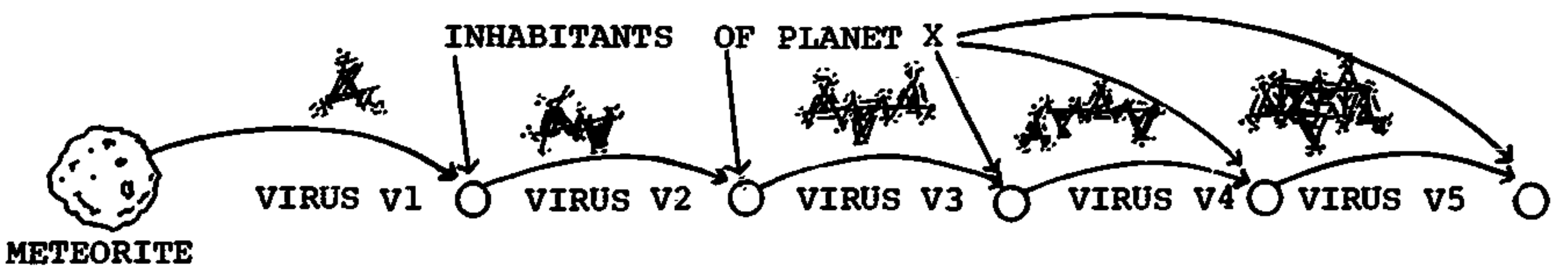
NOW FOR SOME BAD NEWS:

If an inhabitant with disease D1 contacts another inhabitant of X, the virus is transferred in mutated form as V2. The only antidote for V2 is called A2. Similarly, further transfer of the virus produces additional mutations with new antidotes required as follows:



DISEASE	D1	D2	D3	D4	NONE
ANTIDOTE	A1	A2	A3	A4	

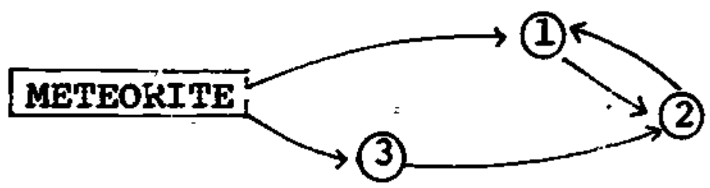
It is possible for an inhabitant to be exposed to several viruses through various contacts, in which case he must receive each appropriate antidote. The amount of antidote to be given is proportional to the number of paths by which the virus was communicated, with 1 gram needed for each communication.

AND NOW FOR SOME GOOD NEWS:

It turns out that virus V5 (and all higher numbered viruses) are harmless. Thus only four antidotes (A1, A2, A3, and A4) are needed to treat any inhabitant of Planet X.

EXAMPLE:

Let's prescribe appropriate antidote dosages for the three inhabitants shown below.



The arrows indicate communication between inhabitants.

$$A^1 = \begin{array}{cc} & \begin{array}{cccc} \boxed{M} & 1 & 2 & 3 \end{array} \\ \begin{array}{c} \boxed{M} \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \qquad A^2 = \begin{array}{cc} & \begin{array}{cccc} \boxed{M} & 1 & 2 & 3 \end{array} \\ \begin{array}{c} \boxed{M} \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

$$A^3 = \begin{array}{cc} & \begin{array}{cccc} \boxed{M} & 1 & 2 & 3 \end{array} \\ \begin{array}{c} \boxed{M} \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array} \qquad A^4 = \begin{array}{cc} & \begin{array}{cccc} \boxed{M} & 1 & 2 & 3 \end{array} \\ \begin{array}{c} \boxed{M} \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

<u>NOTE</u>: Only the first row, (M) should be examined in this problem, since viruses can only <u>originate</u> with the meteorite M.

SOLUTION:

	A1	A2	A3	A4
INHABITANT 1	1 GRAM	0 GRAMS	2 GRAMS	0 GRAMS
INHABITANT 2	0 GRAMS	2 GRAMS	0 GRAMS	2 GRAMS
INHABITANT 3	1 GRAM	0 GRAMS	0 GRAMS	0 GRAMS

(CAN YOU FIND ALL THESE DISEASE COMMUNICATING PATHS?)

<u>NOW BACK TO</u> ...

<u>PROBLEM 4</u>:

Prescribe drugs for the 17 inhabitants of Planet X shown on the cover of this module. If you don't have a computer you have our sympathy!

<u>Some selected answers to the above problems</u>:

1. You should have found a total of <u>18</u> paths from Eichelbergertown to Applebachsville.

2. (b) There are <u>five</u> ways to go from Hawaii to Washington with stops in three cities:

Hawaii	to	Chicago	to	New York	to	Philadelphia	to	Washington
Hawaii	to	Chicago	to	Detroit	to	Philadelphia	to	Washington
Hawaii	to	Chicago	to	Cleveland	to	Pittsburgh	to	Washington
Hawaii	to	Chicago	to	Cleveland	to	Philadelphia	to	Washington
Hawaii	to	Seattle	to	New York	to	Philadelphia	to	Washington

4. The dosage for Inhabitant <u>5</u> is:

   1 gram of A1, 1 gram of A2, 2 grams of A3, 3 grams of A4.